Carnegie Mellon University
21-122, Summer Session 2012

Fourth Homework, due July 30th

1. Use the integral test to decide wether the series converge:

   I) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

   II) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

2. Determine wether the series converges or diverges:

   I) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!}$

   II) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

   III) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n^{\ln n}}$

   IV) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sin(n)}{n}$

3. Find the radius of convergence of the following power series:

   $a)$ $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

   $b)$ $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!}$

   $c)$ $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{(2)(4)(6)\ldots(2n)}$

4. Let $f(x) = 1 + 2x + x^2 + x^4 + \ldots$. Find the interval of convergence of f and an explicit formula for it.

5. Find a power series for $f(x) = \dfrac{x}{1+9x^2}$.

6. Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Find the intervals at which f, f' and f'' converge.

7. Find the Taylor series of e^x centered at $a = 3$.

8. Use series to find $\lim_{x \to 0} \frac{1 - cos(x)}{1 + x - e^x}$.

9. Find the first three nonzero terms in the Maclaurin series of $\frac{x}{\sin(x)}$.